JINSTAR
今之星国际教育

Year 7
Term 2
Lesson One : Averages

Our Learning Objectives

1. To be able to calculate mode, median, mean and range
2. To understand how to use these to compare distributions

Homework Review

Review students homework

Review: Key Points

New concept

I want you to think about the question

I want you to write down an answer

Tell me how you feel about the learning objective or activity

2:00

Task timer

Assess your own work

Teamwork:
Let's work together

Feedback to me/you/each other

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Our Learning Objectives

1. To be able to calculate mode, median, mean and range

What Is an Average?

Other words you often hear to describe an average are 'common' or 'typical'…

A typical number in a set of numerical data.

There are three types of average:

- mode
- mean
- median

To start with, we will introduce the **mode** and also look at the **range**.

The Mode

Look at the hair colours in this group of people.
Which hair colour do you see the most often?

Hair Colour	Tally
Blonde	I
Brown	IIII
Ginger	I

The mode is the one that you see most often.

Brown is the hair colour you see the most often.

Therefore, the mode is brown.

The mode is the only average that can be applied to **non-numerical** data as well as numerical data.

There may be **more than one mode** or **no mode**.

Your Turn

Watch out! There could be more than one mode or no mode at all.

Which colour of bead is the mode?

Red

Which colour of car is the mode?

Silver

Which sort of cupcake is the mode?

No mode. There are 2 of each cupcake.

Which colour of book is the mode?

Grey and green

For each group of numbers below, write down the mode.

5, 4, 3, 5 **5**

5, 3, 0, 7, 3, 7, 8, 2, 3, 7, 0 **3 and 7**

2, 7, 4, 2, 7, 5, 3, 2, 6, 8 **2**

1, 2, 3, 4, 5, 6, 7, 8, 9 **No mode**

Finding the Modal Value

Find the modal number of goals scored.

Goals scored	Frequency
0	5
1	2
2	7
3	0
4	1

A common mistake is to give the answer as 4, as this is the highest amount of goals scored in a match. This is not the mode.

It might help to write the data in a list in ascending order:

0 0 0 0 0 1 1 2 2 2 2 2 2 2 2 4

We can now see that 2 is the modal value.

With numerical data, the mode can be referred to as the modal value.

This is often the case with groups of data.

Take Notes

The easiest way to find the modal value in a table is to look down the frequency column and find the largest value. Then its corresponding value in the first column will be the modal value.

Why Choose the Mode?

There are advantages and disadvantages of selecting the mode as your chosen average.

Disadvantages:

The mode could be much lower or higher than most of the set of data.

If there are too many groups of data which have the same frequency then the mode is not as useful.

If no one piece of data occurs more frequently than any other then the mode cannot be found.

Advantages:

It is the only average that you can use for non-numerical data so it is automatically the most useful in these cases.

It is useful if one item appears a lot more frequently than any other item.

The Range

Find the range in this set of data:
3, 5, 7, 9, 2, 6

It helps to put the numbers in order of size first, starting with the smallest:
2, 3, 5, 6, 7, 9

Now, select the highest and lowest numbers and find the difference between them.
9 − 2 = 7

The range is **7**

The range is the difference between the highest and lowest numbers in a set of data. It allows us to find out how spread out the values are. This is known as the 'measure of spread'.

One way to help you remember:

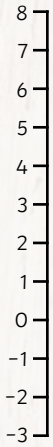
think of a mountain range – you want to find the height difference between the highest and lowest mountains in that range.

The set of data may contain negative numbers too:
5, −3, 4, 2, −1, 8

−3, −1, 2, 4, 5, 8

8 − −3 = **11**

Remember, two minus signs next to each other make a plus.



Your Turn: The Range

The range is the difference between the highest and lowest numbers in a set of data.

Remember to put the numbers in order of size first, starting with the smallest.

Calculate the range for each of these sets of data.

1.	5, 7, 2, 5, 4, 9, 3	2, 3, 4, 5, 5, 7, 9	9 − 2 = **7**
2.	12, 4, 5, 0, 1, 6, 2	0, 1, 2, 4, 5, 6, 12	12 − 0 = **12**
3.	5, −2, 7, 4, −6	−6, −2, 4, 5, 7	7 − −6 = **13**
4.	−3, −1, −5, −2, −8	−8, −5, −3, −2, −1	−8 − −1 = **−7**

Extension question

A 5th number is added to this set of numbers: 3, 7, 9, 3
This number increases the range by 3.

a. Write down two possible values for the 5th number. Show your working.

The current range is 9 − 3 = 6
The new range is 6 + 3 = 9
So the 5th number is either 0 or 12:
12 − 3 = 9 or 9 − 0 = 9

b. State the modal value after the 5th number is added. Will the new number affect the mode? State a reason for your answer.

The modal value is 3.
The 5th number does not affect the mode as there are still two 3s but only one of each of the remaining numbers.

The Median

To find the median, you place the numbers in order of size then find the middle number.

We can use this rhyme to help us remember each average:

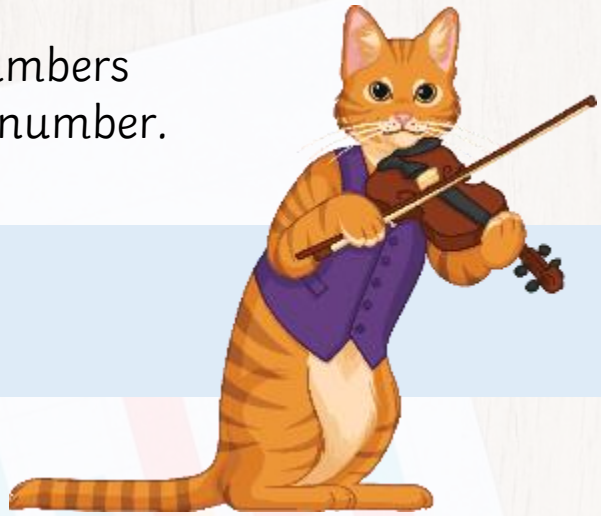
Hey diddle diddle
The median's the middle
You add then divide for the mean
The mode is the one you see the most
And the range is the difference between.

5, 7, 8, 2, 1 → 1, 2, 5, 7, 8 → 1, 2, **5**, 7, 8

5 is in the middle therefore the median is 5.

The Median

To find the median, you place the numbers in order of size then find the middle number.



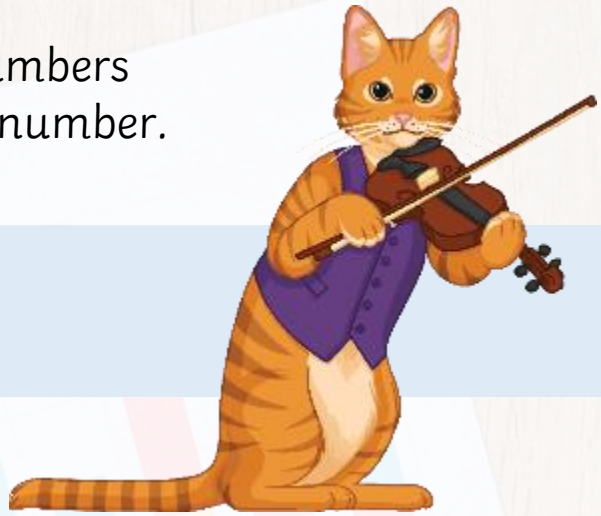
Hey diddle diddle
The median's the middle…

5, 7, 5, 8, 2, 1 → 1, 2, 5, 5, 7, 8 → 1, 2, **5**, **5**, 7, 8

Both the middle numbers are 5 therefore the median is 5.

The Median

To find the median, you place the numbers in order of size then find the middle number.



Hey diddle diddle
The median's the middle…

5, 7, 6, 8, 2, 1 → 1, 2, 5, 6, 7, 8 → 1, 2, **5**, **6**, 7, 8

5 and 6 are both in the middle so we have to find the mid-point between the two numbers.

5 + 6 = 11
11 ÷ 2 = 5.5

The median is **5.5**

Your Turn

Calculate the median for each of these data sets.

1. 3, 4, 5, 6, 7, 8, 9, 10, 13 **7**
2. 11, 12, 13, 15, 18, 20 **14**
3. 5, 4, 1, 9, 8 **1, 4, 5, 8, 9 median = 5**
4. 11, 21, 5, 9 **5, 9, 11, 21 median = 10**

Worded Problems

The table shows the wages a small company pays to its employees each week, in pounds.

Name	Wage
Daniel	292.50
Kalem	320
Zanita	310.50
Janti	325
Rhys	195
Ffion	199.50

Step 1

Put the numbers in order of size, starting with the smallest.

195, 199.50, 292.50, 310.50, 320, 325

Step 2

Find the middle value (or values).

292.50 and 310.50 are the middle values.

Step 3

As we have two middle values, we must calculate the mid-point between the values to find the median.

292.50 + 310.50 = 603

603 ÷ 2 = 301.50

Final answer:

The median average wage the company pays to its employees each week is **£301.50**

Dealing with Large Quantities of Data

Find the median of: 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 4, 4

If we were to cross off the numbers in pairs, this would take a long time and we would probably make a mistake. There are only 40 pieces of data here but if we were dealing with an even larger data set, it would be even more difficult.

We need a shortcut.

Large amounts of data are usually presented in table format:

Data	0	1	2	3	4
Frequency	6	8	7	10	9

Dealing with Large Quantities of Data

Data	0	1	2	3	4	**Total**
Frequency	6	8	7	10	9	**40**

Start by finding the total number of data points.

The total of the frequencies: 6 + 8 + 7 + 10 + 9 = 40

To find the median, first check whether the total is an odd or even number.

If there are an even number of data points, there will be two pieces of data in the middle.

Dealing with Large Quantities of Data

Data	0	1	2	3	4	**Total**
Frequency	6	8	7	10	9	**40**
Cumulative Frequency	6	14	21 (stop)			

the next consecutive piece of data.

For example, 40 ÷ 2 = 20 so the median will be between the 20th and 21st pieces of data.

20th and 21st pieces of data.

In this case, the 20th and 21st pieces of data are both 2 therefore the median is 2.

Dealing with Large Quantities of Data

What if there are an odd number of data points?

Data	0	1	2	3	4	**Total**
Frequency	5	7	9	14	16	**51**
Cumulative Frequency	5	12	21	35 (stop)		

If there are an odd number of data points, there will only be one piece of data in the middle.

Start by adding 1 to the frequency and then divide by two to find this piece of data from the table.

For example, 51 + 1 = 52

52 ÷ 2 = 26 so the median will be the 26th piece of data.

Cumulate (add on) the frequencies until you find the value of the 26th piece of data.

In this case, the 26th piece of data is 3 therefore the median is **3**.

Worded Problems

1. A group of students were asked about the number of mobile electronic devices in their homes. These are the results:

Number of devices	0	1	2	3	4	5	6
Frequency	0	2	5	10	8	4	3

Calculate the median number of devices.

32 pieces of data ÷ 2 = 16th/17th piece of data.
0 + 2 + 5 + 10 = 17
Median = 3

Worded Problems

2. A group of year 11s were asked how many younger siblings they have. The results are shown below.

Number of siblings	0	1	2	3	4	5
Frequency	10	40	130	145	50	25

a. How many younger siblings did the students have in total?

0 × 10 + 1 × 40 + 2 × 130 + 3 × 145 + 4 × 50 + 5 × 25 = 1060

b. Calculate the median number of younger siblings per student surveyed.

400 ÷ 2 = 200th/201st piece of data.
10 + 40 + 130 = 180
10 + 40 + 130 + 145 = 325
median = 3

c. How many students have more than the median number of younger siblings?

50 + 25 = 75

Worded Problems

3. 23 players took part in a basketball competition. The number of baskets scored by each player is recorded in the table below.

Number of baskets scored	Number of Players
0	3
1	6
2	5
3	7
4	2

a. Calculate the median number of baskets scored.

23 + 1 = 24
24 ÷ 2 = 12th piece of data
3 + 6 = 9
3 + 6 + 5 = 14
The median number of baskets scored is 2.

b. A player is selected at random. Is this player more likely to have scored the median or modal number of baskets? Justify your answer.

The modal number of baskets is 3.

The probability of picking a player who scored 2 baskets is $\frac{5}{23}$

The probability of picking a player who scored 3 baskets is $\frac{7}{23}$

It is more likely that the player scored 3 baskets (the modal number).

Exam Questions

James and Mo play cricket for their district. The table below shows the runs they achieved in their previous 6 matches.

James	50	23	39	100	5	15
Mo	25	36	27	40	33	21

a. Calculate the median of both players' runs.

James: 5, 15, 23, 39, 50, 100

23 + 39 = 62

median = 62 ÷ 2 = 31

Mo: 21, 25, 27, 33, 36, 40

27 + 33 = 60

median = 60 ÷ 2 = 30

b. Is the median a good choice of average to determine who is the better batsman? Justify your answer.

The median may not be a good choice here as it ignores the extreme values. James has one very high score and one very low score. Whereas Mo's scores are more consistent.

Both players would need to have scores with a more consistent spread for the median to be a good choice.

The Mean

To find the mean average, we add up all the numbers in the data set and then divide by how pieces of data there are.

Even if all the pieces of data are integers, the mean may not always be an integer as well.

We can use this rhyme to help us remember each average:

Hey diddle diddle
The median's the middle
You add then divide for the mean
The mode is the one you see the most
And the range is the difference between.

The Mean

Calculators may be used for all questions if required.

To calculate the mean average of this data set: 5, 7, 8, 2, 1

We would start by adding up all the pieces of data:

$5 + 7 + 8 + 2 + 1 = 23$

Then, we divide by how many pieces of data there are.

We have 5 pieces of data, so our calculation is $23 \div 5$

Final answer:
The mean average is **4.6**

Your Turn

Calculate the mean average for each of these data sets, leaving your answer correct to one decimal place where appropriate.

1. 3, 4, 5, 6, 7, 8, 9, 10, 13 — 65 ÷ 9 = **7.2**
2. 11, 12, 13, 15, 18, 20 — 89 ÷ 6 = **14.8**
3. 5, 4, 1, 9, 8 — 27 ÷ 5 = **5.4**
4. 11, 21, 5, 9 — 46 ÷ 4 = **11.5**

Worded Problems

The table shows the wages a small company pays to its employees each week, in pounds.

Calculate the mean average weekly wage of an employee at the company, giving your answer in pounds.

Name	Wages
Daniel	293
Kalem	320
Zanita	311
Janti	325
Rhys	195
Ffion	200

$293 + 320 + 311 + 325 + 195 + 200 = 1644$

$\frac{1644}{6} = 274$

The mean average wage is £274.

Worded Problems

1. Here are some results from a science experiment.

   2.5, 3.1, 2.7, 5.0, 3.1, 2.6, 5.8, 3.6

   Calculate the mean average result.

$\frac{28.4}{8} = 3.55$

2. The times below (in minutes) were spent playing on a games console. Calculate the mean average time. Give your answer in minutes and seconds.

   52, 28, 90, 32, 40

$\frac{242}{5} = 48.4$

$0.4 \times 60 = 24$

48 minutes and 24 seconds.

Worded Problems

3. Carrie and Kelsey play darts. Their scores for the last 6 rounds are recorded. Who has the higher mean average score? Round your answers to one decimal place.

   Carrie: 100, 50, 25, 57, 41, 80
   Kelsey: 53, 100, 20, 80, 40, 62

Carrie: $\frac{353}{6} = 58.8$ 1d.p.

Kelsey: $\frac{355}{6} = 59.2$ 1d.p

Kelsey has a higher mean score.

4. Jerri has 4 number cards:

   5, 8, 3 and x

   The mean average of the cards is 5.

   What is the value of x?

$5 + 8 + 3 = 16$

$5 \times 4 = 20$

$20 - 16 = 4$

$x = 4$

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Our Learning Objectives

2. To understand how to use averages to compare distributions

Measures of average - Comparing distributions

Complete the following table. The data shows the scores in two tests, English and Maths, across a set of ten students.

Use the data below to fill in the table.

English	67	73	101	68	66	85	69	86	101	64
Maths	77	78	76	78	78	76	80	79	78	80

	English	Maths
Mean score		
Median score		
Modal score		
Range of scores		

Measures of average - Comparing distributions

Complete the following table. The data shows the scores in two tests, English and Maths, across a set of ten students.

Use the data below to fill in the table.

English	67	73	101	68	66	85	69	86	101	64
Maths	77	78	76	78	78	76	80	79	78	80

	English	Maths
Mean score	78	78
Median score	71	78
Modal score	101	78
Range of scores	37	4

What would each measure of average (and spread) suggest to you about the scores in English and Maths?

- Range

The range of scores in English (37) is far greater than that in Maths (4). Everyone got very similar marks in the Maths test.

- Mean

The mean score in each subject is 78 which implies that the scores of the students are more-or-less identical in English and Maths.

But looking at the actual scores, you can see that this is not the case.

What would each measure of average suggest to you about the scores in English and Maths?

- Median

If you compare the medians (71 and 78), you might assume that the students generally scored less in English. This is partly true, but there are also some much higher scores there too!

- Mode

If you just state the modal score for each subject, you have no information about the scores of the other students. As 101 is higher than 78, you might think that the English scores were all generally higher.

If you were to compare the scores in the two subjects, English and Maths, which measure of average would you use and why?

- It seems that to give maximum information, a **combination of the median and the range** would be good:

In summary, English has a median score of 71 and a range of 37, and Maths has a median score of 78 and a range of 4.

What could you use to back up each of these claims?

- English test results are really high this year
- Students are doing no better in Maths than in English
- People copied each other on the Maths test
- The Maths test was easier than the English test
- The English test results show that some people tried much harder than others

Compare the times of these two groups of athletes.

- 19 runners complete a marathon.
- The times of the 10 professional athletes are (in minutes):
- 133 134 136 139 141 143 144 145 151 158
- The times of the 9 amateur athletes are (in minutes):
- 139 147 151 152 159 161 167 178 182

- Find the mode, median, mean and range of each set of data and use these to compare the times

Compare the times of these two groups of athletes.

- The times of the 10 professional athletes are (in minutes):
- 133 134 136 139 141 143 144 145 151 158
- The times of the 9 amateur athletes are (in minutes):
- 139 147 151 152 159 161 167 178 182

	Professionals	Amateurs
Mean	142.4	159.6
Median	142	159
Mode	None	None
Range	25	43

Lets review

I am able to calculate mode, median, mean and range
I understand how to use these to compare distributions

Homework

Complete the homework sheets sent after the lesson